Examples and Exercises on Forecast Quality Attributes and Metrics

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Plan of lecture

- Example 1: Calculation of the Brier score (exercise 1)
- Example 2: Calculation of the Ranked probability score (exercise 2)
- Example 3: Equatorial Pacific SST prob. forecasts (event positive SST anoms.)
  - Exercise 3: computation of Brier Score and its 3 components in the decomposition, Brier Skill Score, and construction of reliability diagram
  - Exercise 4: Construction of boxplots of forecast probs conditioned on obs. outcomes, ROC curve, and calculation of area under ROC curve
- Example 4: 3 category probability forecasts (Niño-3 index)
  - Exercise 5: Construction of ROC curves and calculation of area under ROC curve for El Niño, Neutral and La Ninã conditions

Seventh WMO International Workshop on Monsoons (IWM-7)
ONLINE TRAINING WORKSHOP ON
SUBSEASONAL TO SEASONAL (S2S) PREDICTION OF MONSOONS
1-12 NOVEMBER 2021

Brier score (BS)

$$BS_j = \frac{1}{n}\sum_{i=1}^{n}\left(y_{j,i} - p_{j,i}\right)^2$$

Brier, G.W., 1950: Verification of forecasts expressed in terms of probability. Monthly Weather Review, 78:1–3.

n: number of forecasts

$\mathbf{y_{i,j}}$: is 1 if the i^{th} observation was in category j, and is 0 otherwise (binary observations)

$\mathbf{p_{i,j}}$: is the i^{th} forecast probability for category j

Interpretation: Average squared "error" in the probability forecasts

Range: from 0 for perfect forecasts (100% probability assigned to the observed category on each forecast) to 1 for perfectly bad forecasts (0% probability assigned to the observed category on each forecast).

The Brier score has to be calculated to each category individually

The Brier score measures reliability and resolution together (as well as uncertainty)

Exercise 1: Brier score calculation for above-normal category probability forecasts

i	$y_{3,i}$ (Binary obs)	$p_{3,i}$ (Forecast prob.)	$(y_{3,i} - p_{3,i})^2$
1	0.0	0.20	0.0400
2	0.0	0.20	0.0400
3	0.0	0.25	0.0625
4	0.0	0.33	0.1111
5	0.0	0.40	0.1600
6	0.0	0.45	0.2025
7	1.0	0.45	0.3025
8	1.0	0.35	0.4225
BS= $\frac{1}{n}\sum_{i=1}^{n}(y_{3,i} - p_{3,i})^2$			0.1676

Ranked probability score (RPS)

$$RPS = \frac{1}{n(m-1)} \sum_{i=1}^{n} \sum_{k=1}^{m-1} \left(\sum_{j=1}^{k} \left(y_{j,i} - p_{j,i} \right) \right)^2$$

(Epstein, 1969, Murphy 1969, 1970, 1971)

n: number of forecasts

m: number of categories

$\mathbf{y_{i,j}}$: is 1 if the i^{th} observation was in category j, and is 0 otherwise

$\mathbf{p_{i,j}}$: is the i^{th} forecast probability for category j

Interpretation: Average squared "error" in the cumulative probabilistic forecasts

Range: from 0% for perfect forecasts (100% probability assigned to the observed category on each forecast) to a maximum of 100% only achieved if all the observations are in the outermost categories, and if the forecasts are perfectly bad forecasts (100% probability assigned to the opposite outermost category to that observed).

The summation over *j* is to *m*-1 rather than to *m* because the cumulative forecasts and observations over all categories are both 100%.

Exercise 2: Ranked probability score calculation for tercile probability forecasts

i	$y_{1,i}$	$p_{1,i}$	$(y_{1,i}-p_{1,i})^2$	$\sum_{j=1}^{2} y_{j,i}$	$\sum_{j=1}^{2} p_{j,i}$	$\left(\sum_{j=1}^{2}(y_{j,i}-p_{j,i})\right)^2$
1	1.0	0.45	0.3025	1.0	0.80	0.0400
2	1.0	0.50	0.2500	1.0	0.80	0.0400
3	1.0	0.35	0.4225	1.0	0.75	0.0625
4	1.0	0.33	0.4444	1.0	0.67	0.1111
5	0.0	0.25	0.0625	1.0	0.60	0.1600
6	0.0	0.20	0.0400	1.0	0.55	0.2025
7	0.0	0.20	0.0400	0.0	0.55	0.2025
8	0.0	0.25	0.0625	0.0	0.65	0.4225
		$\frac{1}{n}\sum_{i=1}^{n}(y_{1,i}-p_{1,i})^2$	0.2031	$\frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{k}(y_{j,i}-p_{j,i})\right)^2$		0.1551
			RPS= $\frac{1}{n(m-1)}\sum_{i=1}^{n}\sum_{k=1}^{m-1}\left(\sum_{j=1}^{k}(y_{j,i}-p_{j,i})\right)^2$			0.1791

RPS references

Epstein, E.S., 1969: A scoring system for probability forecasts of ranked categories. Journal of Applied Meteorology, 8:985–987.

Murphy, A.H., 1969: On the “ranked probability score”. Journal of Applied Meteorology, 8:988–989.

Murphy, A.H., 1970: The ranked probability score and the probability score: A comparison. Monthly Weather Review, 98:917–924.

Murphy, A.H., 1971: A note on the ranked probability score. Journal of Applied Meteorology, 10:155–156.

Skill Scores

The scores are often presented as skill scores (SS) by using the linear transformation:

$$SS = \frac{S - S_{ref}}{S_{best} - S_{ref}} = \frac{S - S_{ref}}{0 - S_{ref}} = 1 - \frac{S}{S_{ref}}$$

where S is the score (e.g. the Brier Score) and S_{ref} is the value of the score for some *unskilful* reference forecast such as:

- Issuing the same constant probability each time
- Issuing random probabilities each time

Note: both of these can be thought of as sampling a probability from a distribution (constant probability is a special limit of zero width distribution)

Skill scores allow easy interpretation of forecasts:

SS = 0 (when $S = S_{ref}$) → no skill forecast
SS = 1 (when $S = S_{best} = 0$) → perfect forecast

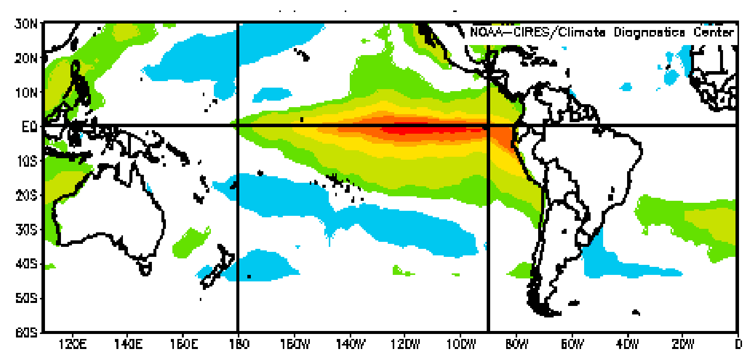
$BSS = 1 - BS/BS_{clim}$

$RPSS = 1 - RPS/RPS_{clim}$

Example 3: Equatorial Pacific SST

88 seasonal probability forecasts of binary SST anomalies at 56 grid points along the equatorial Pacific. Total of 4928 forecasts.

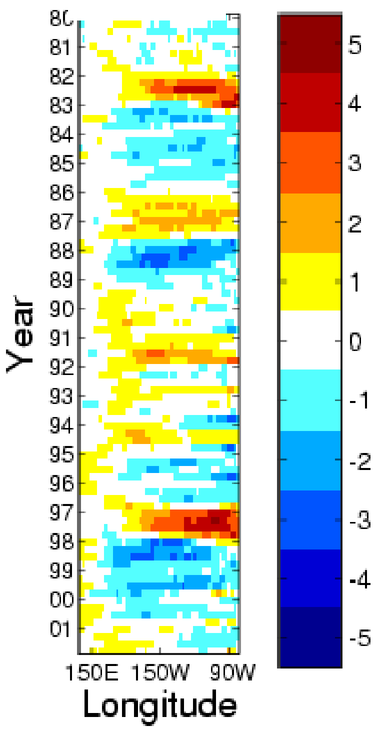
6-month lead forecasts for 4 start dates (F,M,A,N) valid for (Jul,Oct,Jan,Aug)



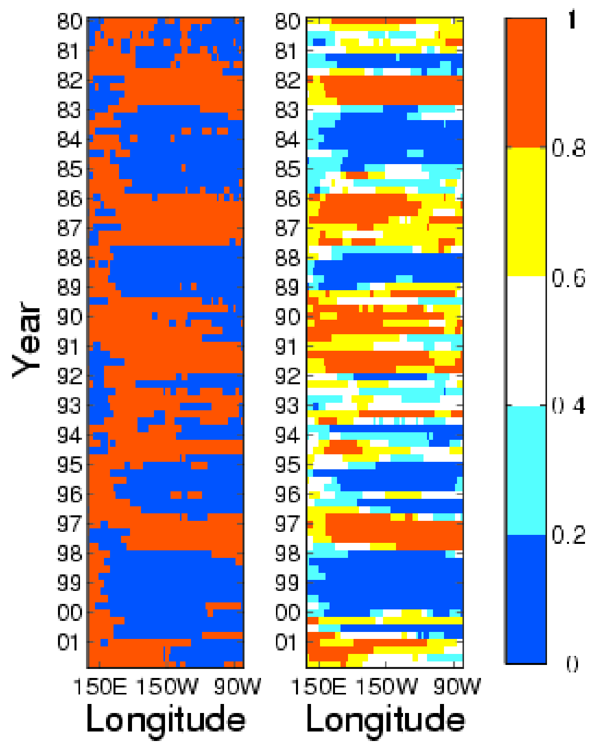
SST

$o = (SST > 0)$ $\quad f = \Pr(\hat{o})$

OBS



OBS ENS



The probability forecasts were constructed by fitting Normal distributions to the ensemble mean forecasts from the 7 DEMETER coupled models, and then calculating the area under the normal density for SST anomalies greater than zero.

SST anomalies (°C)

Forecast probabilities: f

Exercise 3:

Read data file equatorialpacificsst.txt which contains forecast probabilities for the event Eq. Pac. SST>0 and the corresponding binary observations

data<-read.table("equatorialpacificsst.txt")

#1st column contains forecast probabilities
probfcsts<-data[,1]

#2nd column contains binary observation
binobs<-data[,2]

```
#Compute the climatological frequency of the event
 obar<-mean(binobs)

#Compute the Brier score for the climatological frequency
#(i.e. the climatological forecast)
bsclim<-mean((obar-binobs)^2)

#Compute the variance of binary observation
var(binobs) *(length(binobs)-1)/length(binobs)

#Compute the uncertainty component of the Brier score
obar*(1-obar)
#How does this compare with the Brier score computed
#above? What can you conclude about the reliabilty and
#resolution components of the Brier score for the
#climatological forecast?
```

```
#Compute the Brier score for the SST prob. forecasts
#for the event SST>0
bs<-mean((probfcsts-binobs)^2)

#How does this compare with the Brier score for the
#climatological forecast? What can you conclude about the
#skill of these forecasts (i.e. which of the two are more
#skillfull by looking at their Brier score values)?

#Compute the Brier skill score
bss <- 1-(bs/bsclim)

#How do you interpret the Brier skill score obtained
#above? I.e. what can you conclude about the skill of the SST
#prob. forecasts when compared to the climatological
#forecast?
```

```
#Use the verification package to compute the Brier score and
#its decomposition for the SST prob. forecasts for
#the event SST>0
library(verification)
A<-verify(binobs,probfcsts, frcst.type="prob",obs.type="binary"
summary(A)

#Note: Brier score – Baseline is the Brier score for the
#reference climatological forecast
#Skill Score is the Brier skill score
#Reliability, resolution and uncertainty are the three
#components of the Brier score decomposition

#What can be conclude about the quality of these forecasts
#when compared with the climatological forecasts?
```

```
#Construct the reliability diagram for these forecasts using
#10 bins
nbins<-10
bk<-seq(0,1,1/nbins)
h<-hist(probfcsts,breaks=bk,plot=F)$counts
g<-hist(probfcsts[binobs==1],breaks=bk,plot=F)$counts
obari <- g/h
yi <- seq((1/nbins)/2,1,1/nbins)

x11()
par(pty='s',las=1)
reliability.plot(yi,obari,h,titl="10 bins",legend.names="")
abline(h=obar)

#What can you conclude about these forecasts by examining
#the feature of the reliability diagram curve?
```

```
# Compute reliability, resolution and uncertainty components
# of the Brier score
n<-length(probfcsts)
reliab <- sum(h*((yi-obari)^2), na.rm=TRUE)/n
resol <- sum(h*((obari-obar)^2), na.rm=TRUE)/n
uncert<-obar*(1-obar)
bs<-reliab-resol+uncert

#How does the results above compare with those obtained
#with the verify function?
```

ROC: Relative operating characteristics

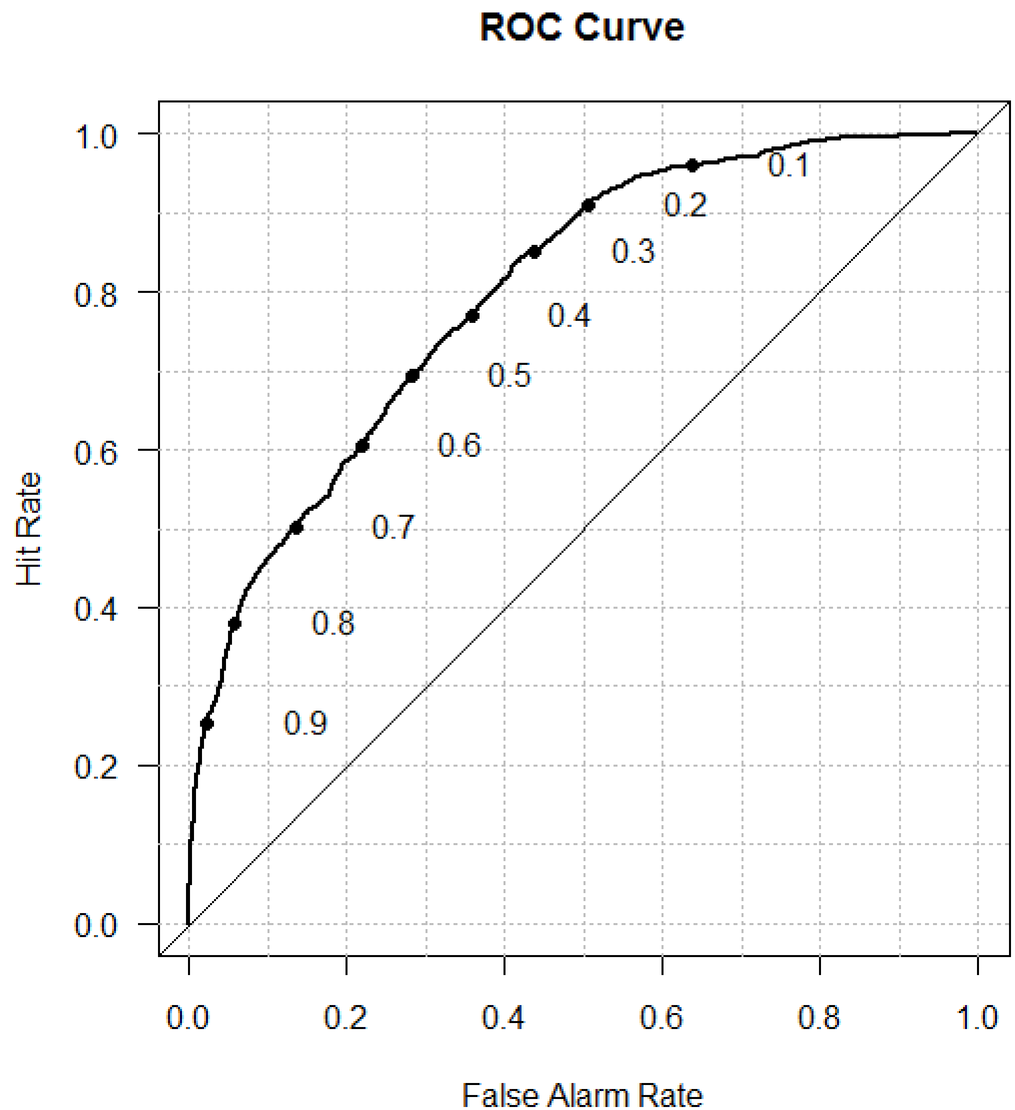
Measures discrimination (ability of forecasting system to detect the event of interest)

Forecast	Observed		
	Yes	No	Total
Yes	a (Hit)	b (False alarm)	a+b
No	c (Miss)	d (Correct rejection)	c+d
Total	a+c	b+d	a+b+c+d=n

Hit rate=a/(a+c)

False alarm rate=b/(b+d)

ROC curve: plot of hit versus false-alarm rates for decreasing prob. thresholds



- The ROC curve is constructed by calculating the hit and false-alarm rates for decreasing probability thresholds
- Area under ROC curve (A) is a measure of discrimination: A = 0.79 (prob. of successfully discriminating a warm (SST>0) from a cold (SST<0) event)

Exercise 4:

Read data file equatorialpacificsst.txt which contains forecast probabilities for the event Eq. Pac. SST>0 and the corresponding binary observations

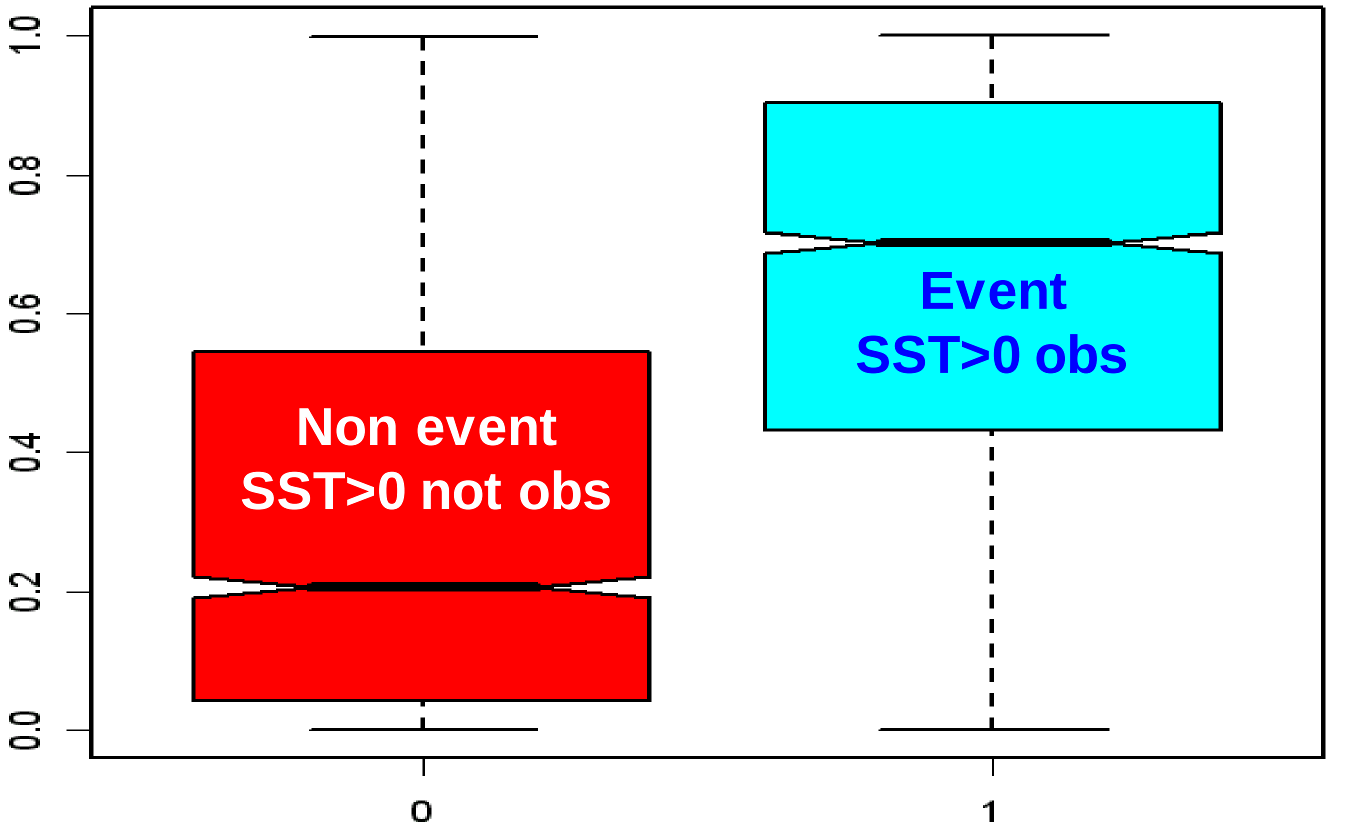
data<-read.table("equatorialpacificsst.txt")

#1st column contains forecast probabilities

#2nd column contains binary observations

Prob. forecasts conditioned/stratified on observations

Forecast probability Pr(SST>0)



Observed binary event X

→ Forecasts do differ given different outcomes

→ Forecast system has discrimination (distinguish event from non-event)

Reproducing the previous plot

1) Stratify forecast probabilities p (1st column of data) on observed (1) and not observed (0) binary events (2nd column od data)

d1 #object containing strat of p on not observed event

```
> d1<-data[data[,2]==0,1]
```

d2 #object containing strat of p on observed event

```
> d2<-data[data[,2]==1,1]
```

2) Produce a boxplot using the command

```
> boxplot(d1,d2,col=c(2,5),notch=T,names=c(0,1))
```

```
# extract only forecast/obs pairs with p >=0.9
p<-0.9
# forecast events
f<-data[data[,1]>=p,]
a<-sum(f[,2]==1) #forecast and observed (hit)
b<-sum(f[,2]==0) #forecast and not observed (false alarm)
# not forecast events
g<-data[data[,1]<p,]
c<-sum(g[,2]==1) #not forecast and observed (miss)
d<-sum(g[,2]==0) #not fcst and not obs (correct rejection)
n<-a+b+c+d
hr<-a/(a+c)
far<-b/(b+d)
```

```
#Plot first point of the ROC curve
par(pty='s',las=1)
plot(far,hr,type="p",pch=16,xlim=c(0,1),ylim=c(0,1),xlab="False alarm rate",ylab="Hit rate")
abline(0,1)
```

```
#repeat the same procedure for p>=0.8

#extract only forecast/obs pairs with p >=0.8
p<-0.8
# forecast events
f<-data[data[,1]>=p,]
a<-sum(f[,2]==1) #forecast and observed (hit)
b<-sum(f[,2]==0) #forecast and not observed (false alarm)
# not forecast events
g<-data[data[,1]<p,]
c<-sum(g[,2]==1) #not forecast and observed (miss)
d<-sum(g[,2]==0) #not fcst and not obs (correct rejection)
n<-a+b+c+d
hr<-a/(a+c)
far<-b/(b+d)
```

```
#Plot new point in the ROC curve
points(far,hr,pch=16)

#repeat the same procedure for p>=0.7, p>=0.6, p>=0.5,
#p>=0.4, p>=0.3, p>=0.2 and p>=0.1 adding the new points
#in the ROC curve. Try later to do this using a for loop.

#The area below the curve that joins all points (the ROC
#area) is a forecast quality measure of discrimination.
#ROC area values equal 0.5 indicate no discrimination.
#ROC area values equal to 1 indicate perfect discrimination.
#ROC area values equal to 0 indicate perfectly bad
#discrimination.
```

```
#Constructing the empirical ROC curve

#find unique forecast probability values
p<-unique(data[,1])
#sort unique fcst prob values from largest to smallest
p<-rev(sort(p))
#define vectors to store hit and false-alarm rates
hr<-rep(NA,length(p)+2)
far<-rep(NA,length(p)+2)
#set first and last point in the ROC curve to (0,0) and (1,1)
hr[1]<-0
far[1]<-0
hr[length(p)+2]<-1
far[length(p)+2]<-1
```

```
#compute hit and false alarm rates for all fcst prob thresholds
for (i in 1:length(p)){
f<-data[data[,1]>=p[i],] #forecast events
a<-sum(f[,2]==1) #hit
b<-sum(f[,2]==0) #false alarm
g<-data[data[,1]<p[i],] # not forecast events
c<-sum(g[,2]==1) #miss
d<-sum(g[,2]==0) #correct rejection
hr[i+1]<-a/(a+c)
far[i+1]<-b/(b+d)
}
#plot empirical ROC curve
par(pty='s',las=1)
plot(far,hr,type="l",xlim=c(0,1),ylim=c(0,1),xlab="False alarm
rate",ylab="Hit rate")
abline(0,1)
```

```
#plot roc curve with verification package for comparison
x11()
roc.plot(data[,2],data[,1])

#compute area under empirical ROC curve
dif<-diff(far)
area<-sum(0.5*(hr[1:((length(hr)-1))]+hr[2:length(hr)])*dif)

#compute ROC area using the verification package
roc.area(data[,2],data[,1])$A

#The ROC skill score is defined as (2*ROC area)-1
#so that positive values indicate better discrim. ability
#and negative values indicate worse discrim. ability than the
#reference 50% (random, guessing) prob. of successfully
#discriminating events from non events
rss<-2*area-1
```

Example 4: 3 category probabilistic forecasts

October NIÑO3 forecasts from five DEMETER models produced in the previous May, together with observed anomalies for the 20 years, 1982-2001. The observed NIÑO3 anomalies are indicated in column 2 and classified as El Niño (E), neutral (N), and La Niña (L).

•The forecasts are presented as probabilities based on a simple count of five DEMETER models.
• Sum of fcts probs E+N+L is 100%.
• Forecast probabilities for each category are assessed separately (i.e. each column of forecast probabilities for El Niño, Neutral and La Niña is assessed separately).
• Each fcst probability column is compared to obs column

			Probabilities of October NIÑO3 produced in the previous May		
Year	Observed		El Niño (E)	Neutral (N)	La Niña (L)
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Exercise 5: Construct ROC curves and calculate the area under ROC curve for El Niño, Neutral and La Ninã conditions

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
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1995	-0.86	L	0	40	60
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1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

El Niño conditions

Observed El Ninos: 5

Non Obs El Ninos: 15

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$0

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥0

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
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1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$0

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
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1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob $\geq$0

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

F =15/15=1

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
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1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥20

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$20

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =4/5=0.8

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥20

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =4/5=0.8

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

F =5/15=0.33

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$40

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$40

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =4/5=0.8

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥40

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =4/5=0.8

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

F =3/15=0.2

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥60

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥60

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =3/5=0.6

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥60

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =3/5=0.6

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

F =0/15=0

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥80

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$80

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =3/5=0.6

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob$\geq$80

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =3/5=0.6

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

F =0/15=0

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥100

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥100

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

H =2/5=0.4

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed El Ninos: 5

Non Obs El Ninos: 15

Prob≥100

$$H = \frac{\text{Hits}}{\text{Observed El Ninos}}$$

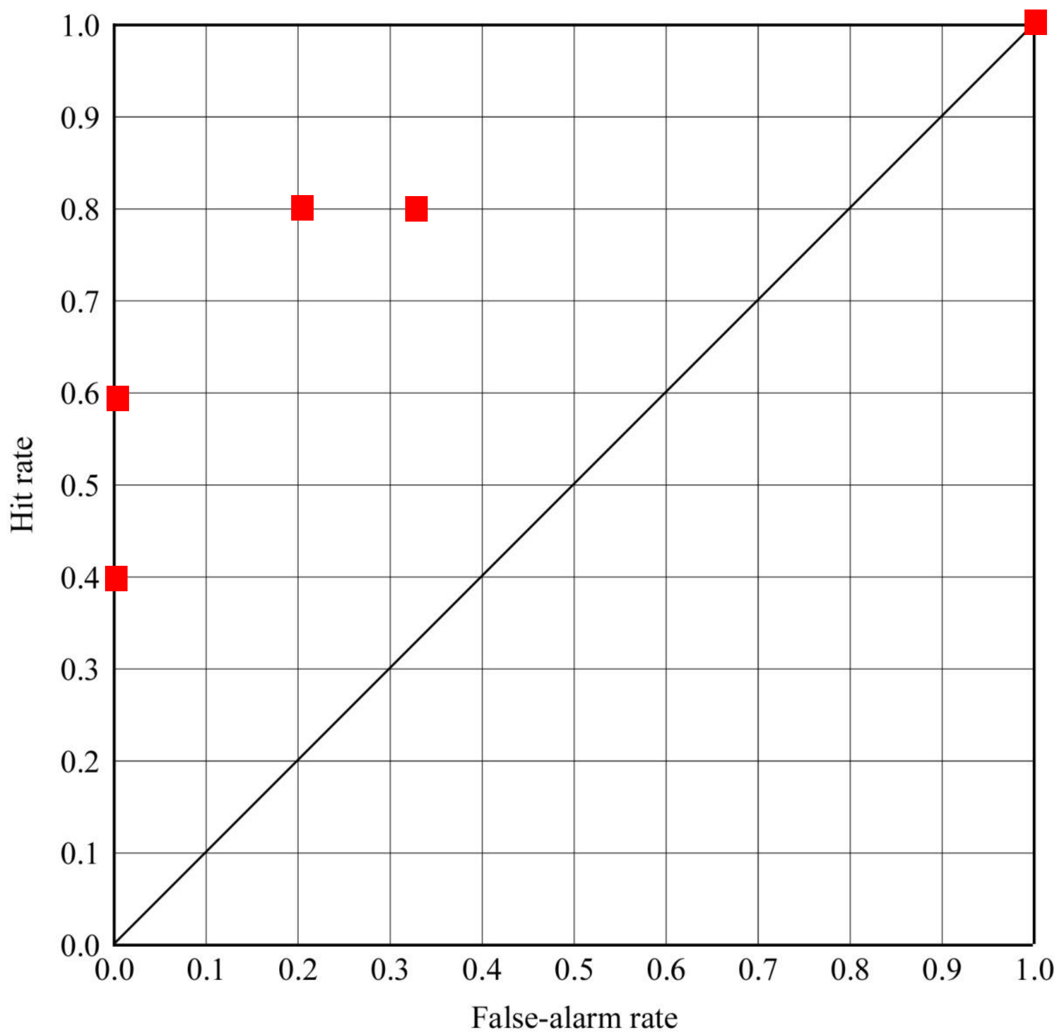
H =2/5=0.4

$$F = \frac{\text{False alarms}}{\text{Non Obs El Ninos}}$$

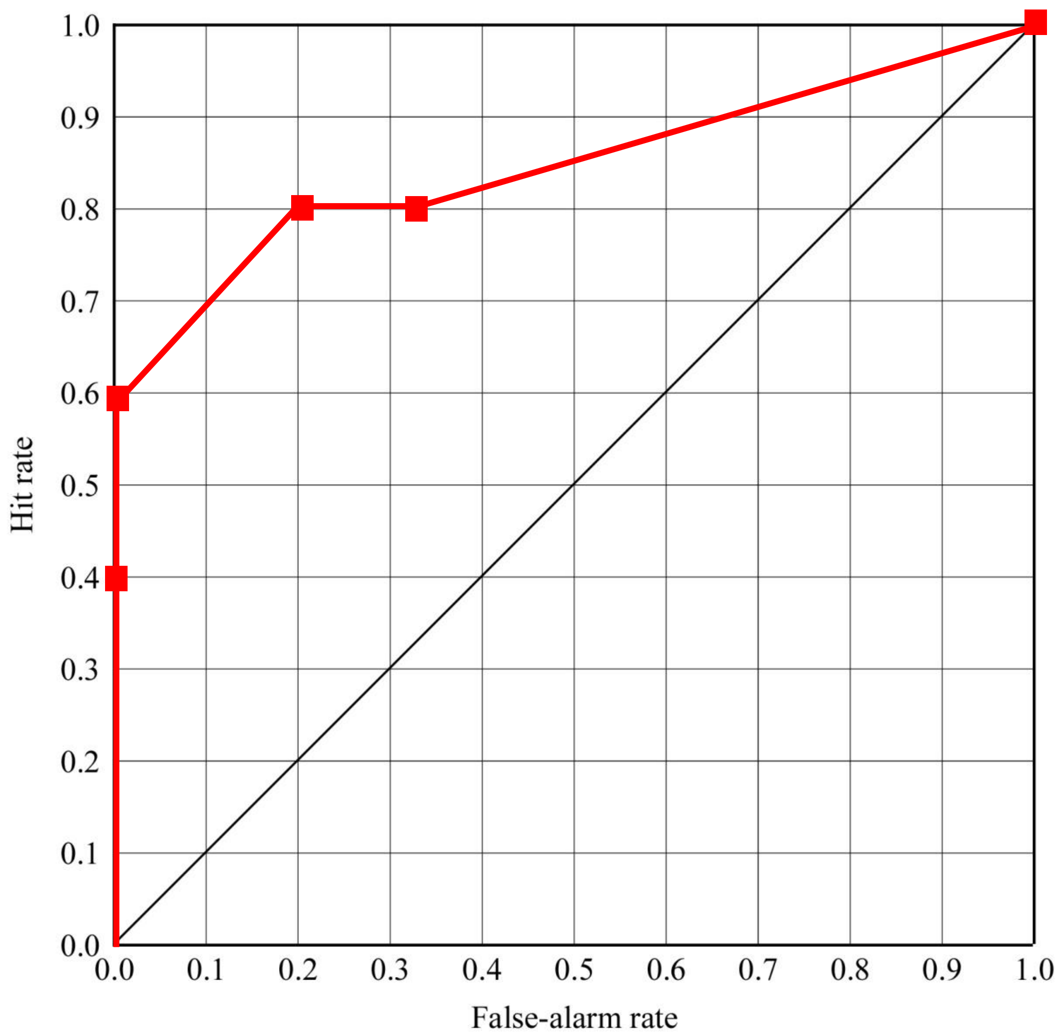
F =0/15=0

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

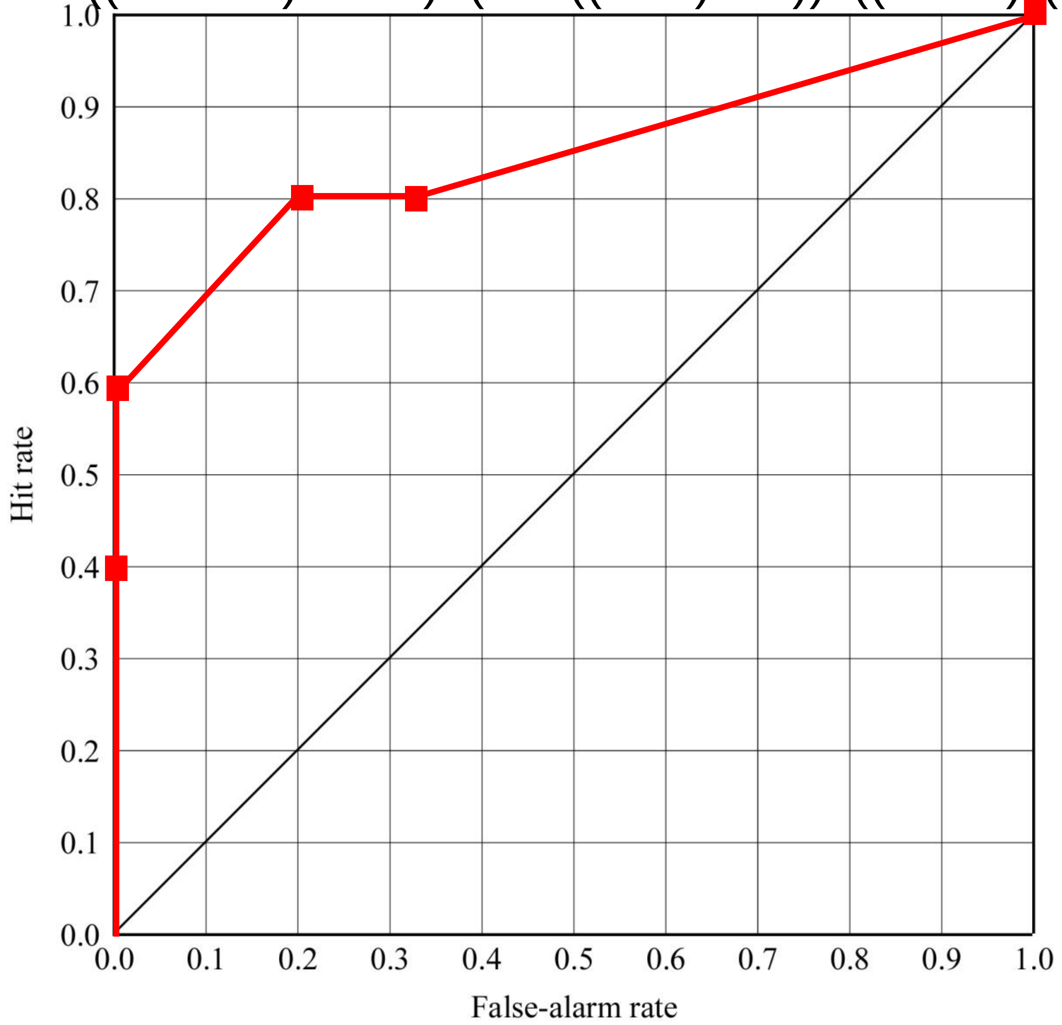


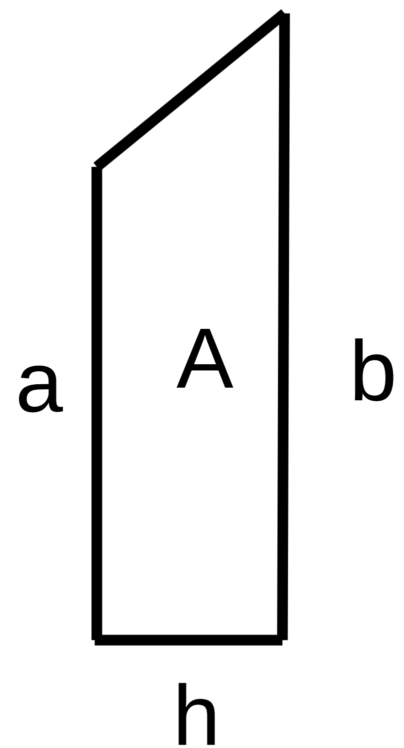
Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1



Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

ROC area=((0.8+0.6)*0.2/2)+(0.8*((5/15)-0.2))+((1+0.8)*(1-(5/15))/2)=0.85





Area A = (a+b)*h/2

Neutral conditions

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥0

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

H =10/10=1

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

F =10/10=1

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥20

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

H =5/10=0.5

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

F =5/10=0.5

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥40

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

H =2/10=0.2

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

F =3/10=0.3

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥60

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

H =1/10=0.1

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

F =0/10=0

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥80

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

H =1/10=0.1

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

F =0/10=0

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed Neutral: 10

Non Obs Neutral: 10

Prob≥100

$$H = \frac{\text{Hits}}{\text{Observed Neutral}}$$

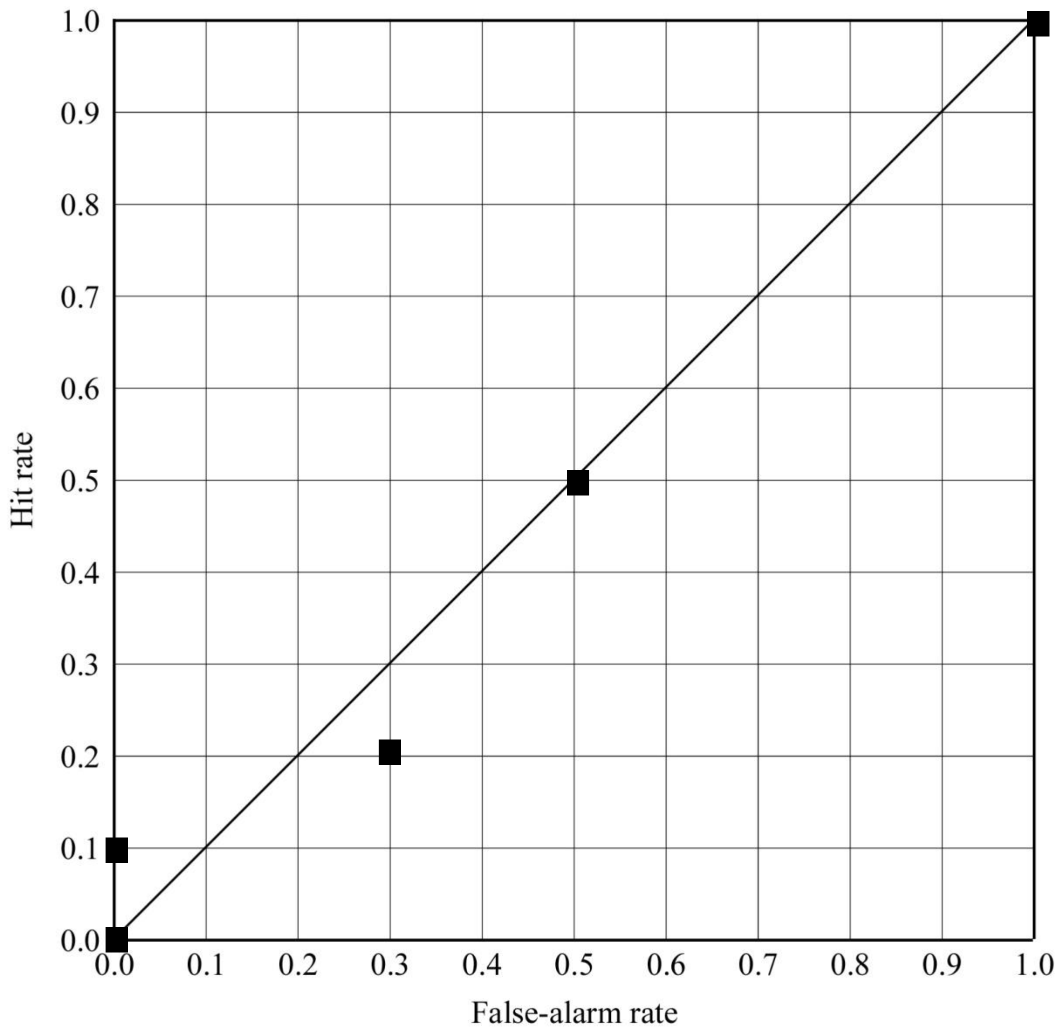
H =0/10=0

$$F = \frac{\text{False alarms}}{\text{Non Obs Neutral}}$$

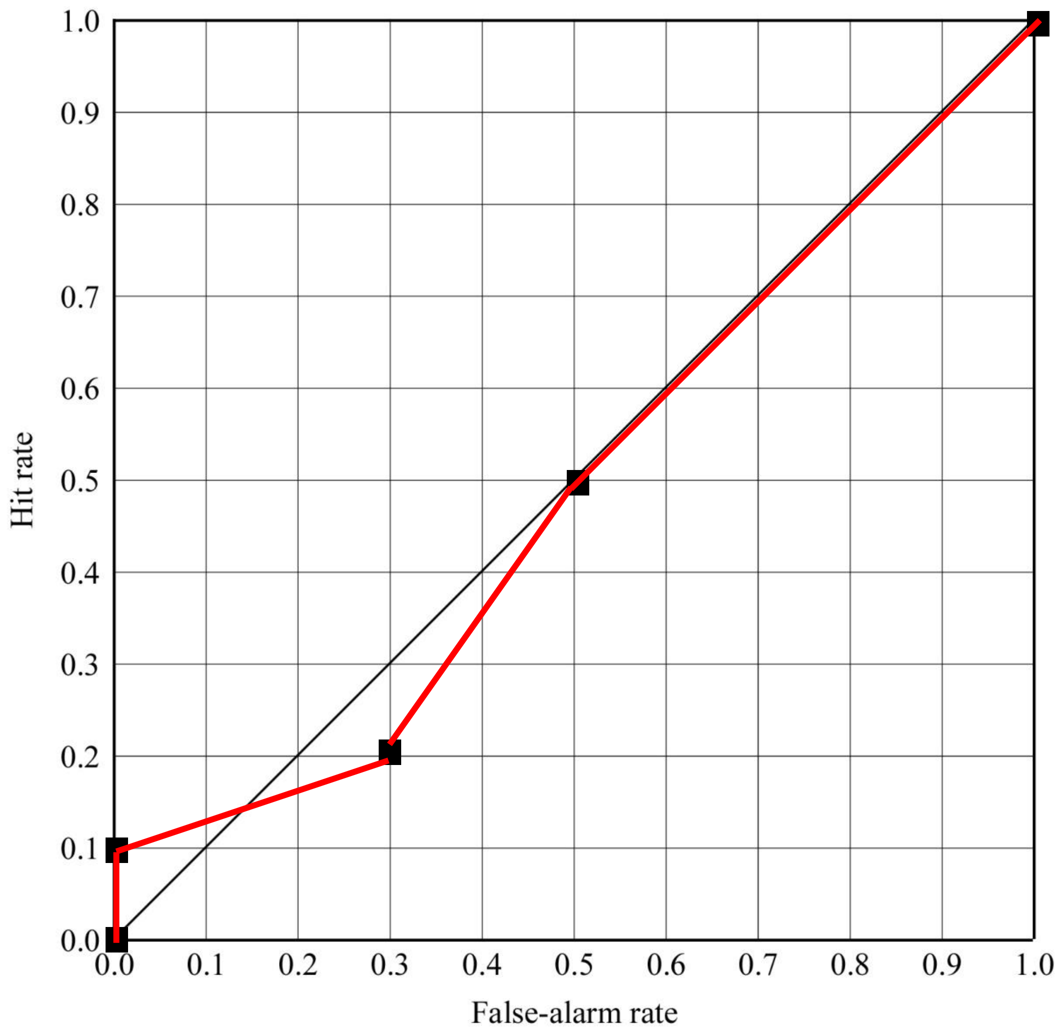
F =0/10=0

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

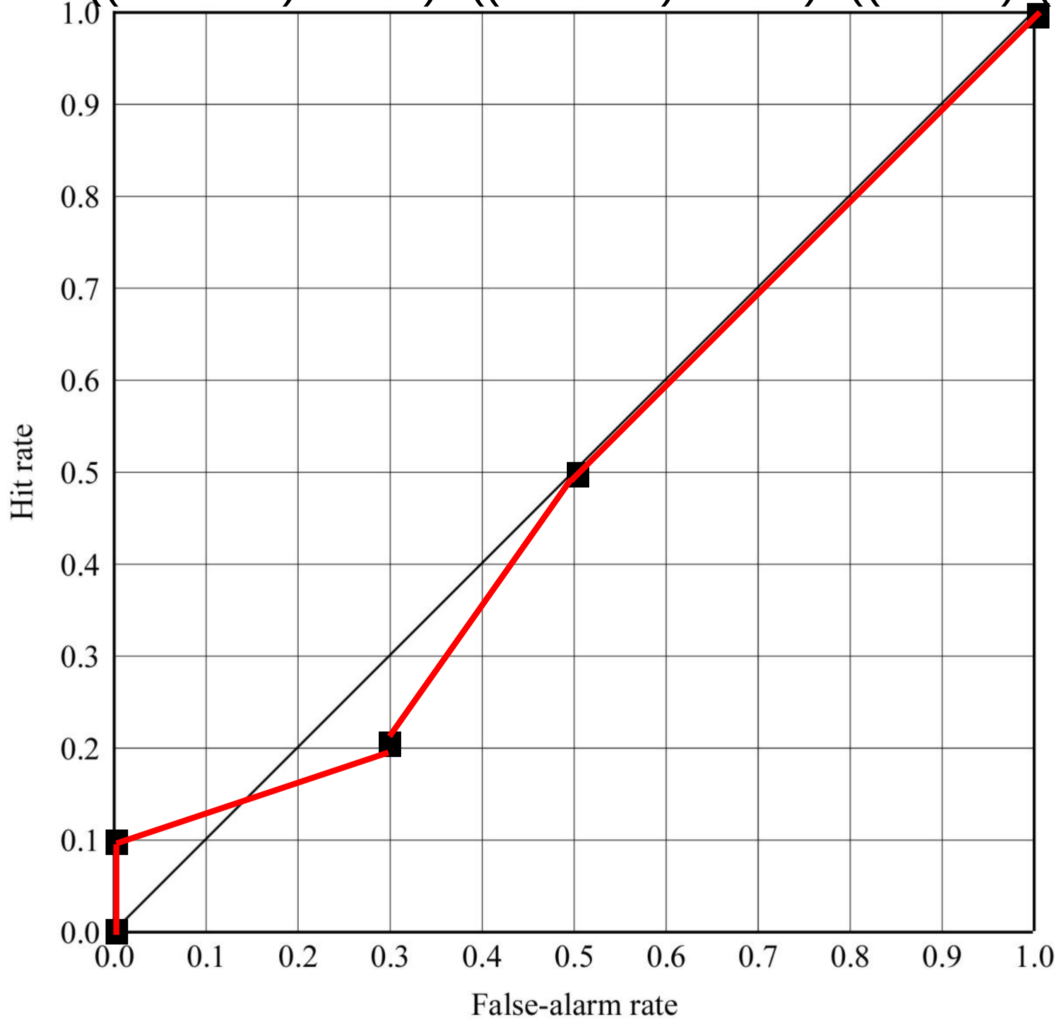


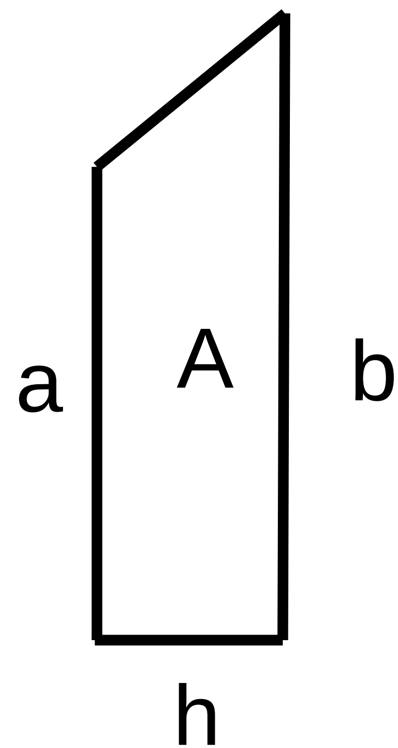
Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1



Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

ROC area=((0.2+0.1)*0.3/2)+((0.5+0.2)*0.2/2)+((1+0.5)*(0.5/2))=0.49





Area A = (a+b)*h/2

La Niña conditions

Observed La Nina: 5

Non Obs La Nina: 15

Prob≥0

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

F =15/15=1

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed La Nina: 5

Non Obs La Nina: 15

Prob≥20

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

F =13/15=0.86

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed La Nina: 5

Non Obs La Nina: 15

Prob≥40

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

F =11/15=0.73

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed La Nina: 5

Non Obs La Nina: 15

Prob≥60

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

H =5/5=1

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

F =10/15=0.66

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed La Nina: 5

Non Obs La Nina: 15

Prob≥80

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

H =2/5=0.4

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

F =6/15=0.4

			Probabilities of October NIÑO3		
Year	Observed		El Niño	Neutral	La Niña
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Observed La Nina: 5

Non Obs La Nina: 15

Prob$\geq$100

$$H = \frac{\text{Hits}}{\text{Observed La Nina}}$$

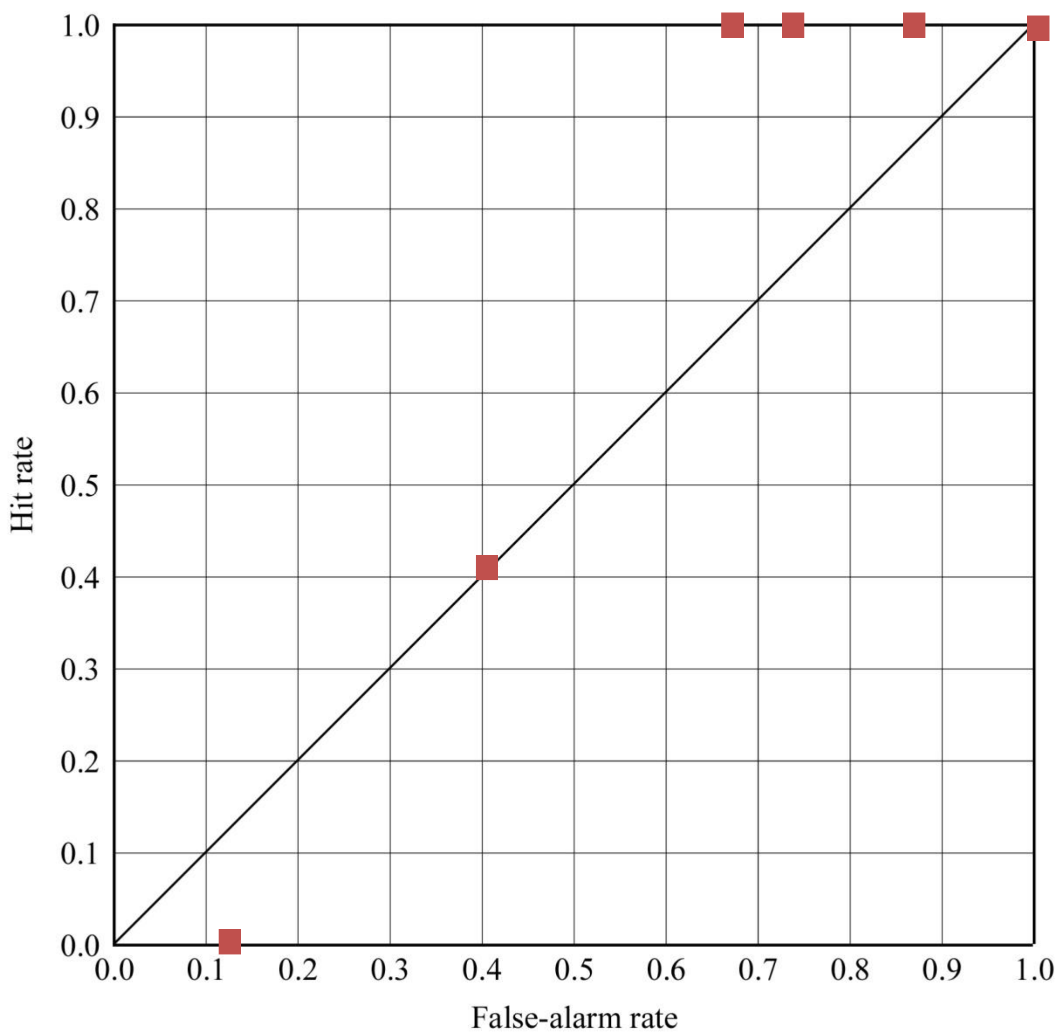
H =0/5=0

$$F = \frac{\text{False alarms}}{\text{Non Obs La Nina}}$$

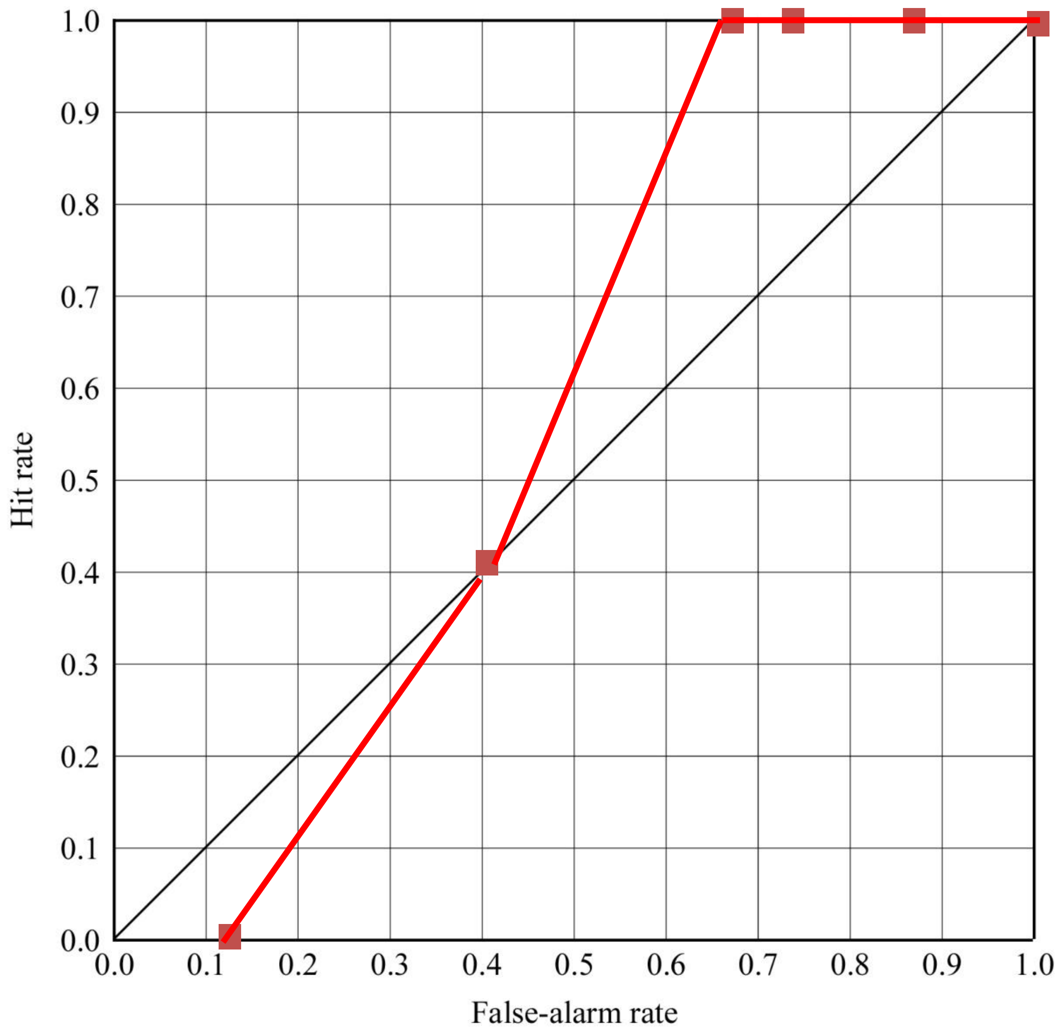
F =2/15=0.13

			Probabilities of October NIÑO3		
Year	**Observed**		**El Niño**	**Neutral**	**La Niña**
1981	-0.23	N	0	40	60
1982	2.07	E	100	0	0
1983	-0.21	N	0	0	100
1984	-0.84	L	0	20	80
1985	-0.82	L	0	20	80
1986	0.55	E	0	0	100
1987	1.28	E	80	0	20
1988	-1.62	L	0	40	60
1989	-0.41	N	20	0	80
1990	-0.10	N	40	20	40
1991	0.62	E	40	0	60
1992	-0.33	N	40	0	60
1993	0.24	N	40	0	60
1994	0.47	N	0	20	80
1995	-0.86	L	0	40	60
1996	-0.49	N	0	20	80
1997	3.02	E	100	0	0
1998	-0.71	N	0	80	20
1999	-1.09	L	0	40	60
2000	-0.54	N	20	0	80

Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

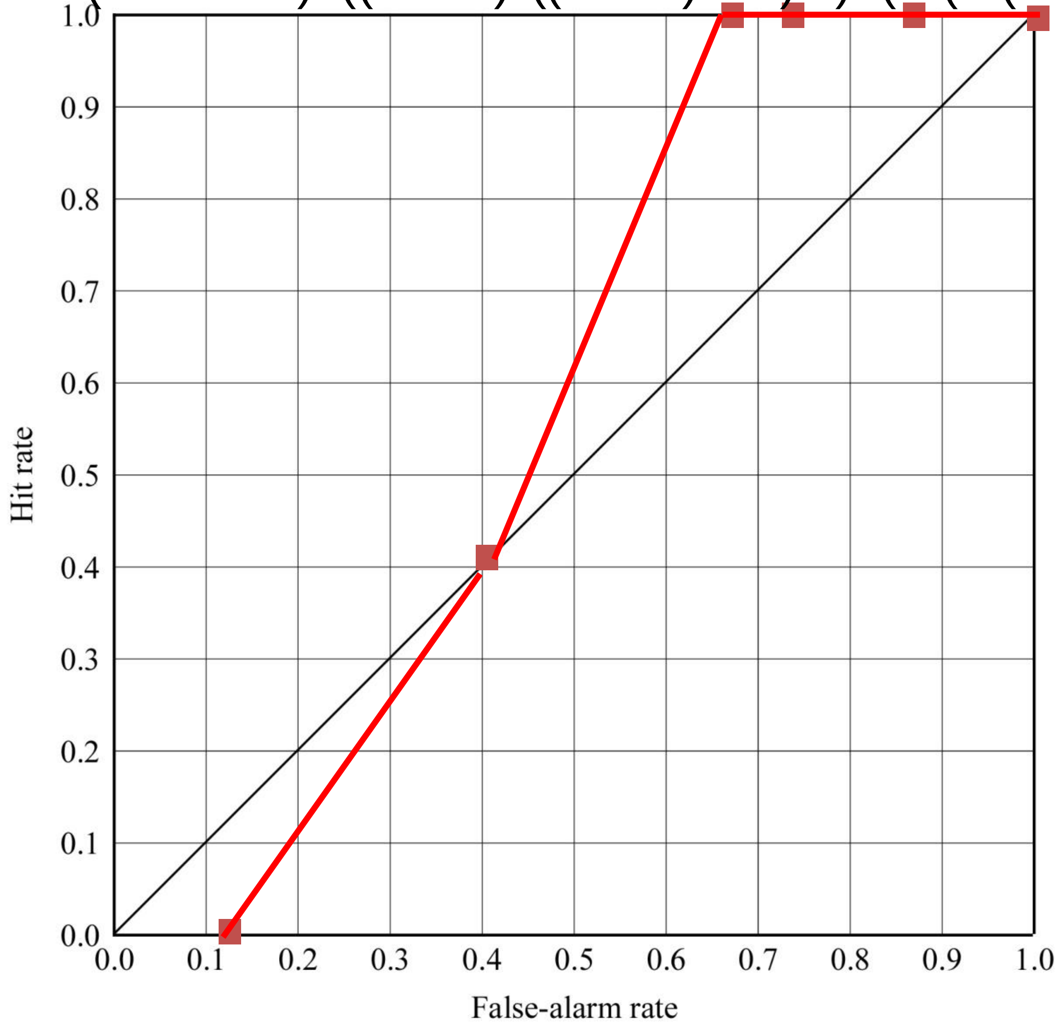


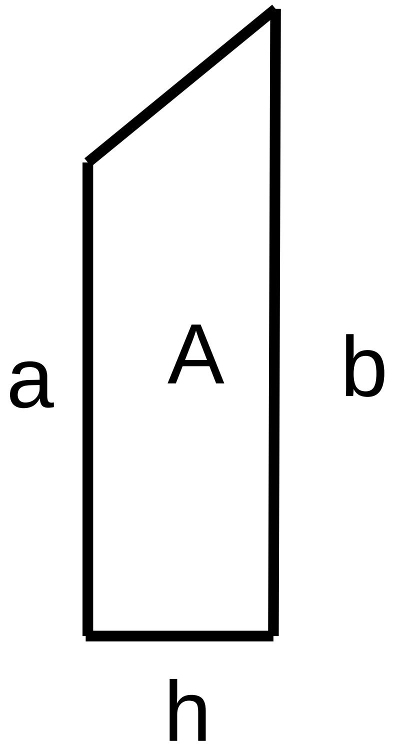
Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1



Warning	El Niño		Neutral		La Niña	
	H	F	H	F	H	F
≥ 100%	2/5=0.4	0/15=0	0/10=0	0/10=0	0/5=0	2/15=0.13
≥ 80%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	2/5=0.4	6/15=0.4
≥ 60%	3/5=0.6	0/15=0	1/10=0.1	0/10=0	5/5=1	10/15=0.66
≥ 40%	4/5=0.8	3/15=0.2	2/10=0.2	3/10=0.3	5/5=1	11/15=0.73
≥ 20%	4/5=0.8	5/15=0.33	5/10=0.5	5/10=0.5	5/5=1	13/15=0.86
≥ 0%	5/5=1	15/15=1	10/10=1	10/10=1	5/5=1	15/15=1

ROC area=(0.4*0.3/2)+((1+0.4)*((10/15)-0.4)/2)+(1*(1-(10/15)))=0.58





Area A = (a+b)*h/2